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CUSP CATASTROPHE MODEL OF CLIMATE CHANGE – COMPANION TO BASIC GLOBAL WARMING

This article examines climate change as an interdisciplinary exposition of economic dynamics, human population dynamics, thermodynamics and nonlinear dynamics. The substance of the writing is about the application of the cusp catastrophe model as characterized by Thom (1975) to climate change. The cusp model under consideration has three global variables: 1) human population, 2) CO_2 emissions and 3) temperature. The relationship at different intersections of the cusp catastrophe model are examined in order to portray a reasonable representation of climate transition. While the article is theoretical in nature and it borrows from the physical sciences to explain sudden change.

Keywords: *climate change, CO2, population, cusp catastrophe, energy systems.*

INTRODUCTION

This article is a continuation of a series on global warming. The first (Jacobsen, 2018) was a study of population, CO_2 and temperature over time. The hypothesis was that there is a positive significant relationship between CO_2 and Earth temperature. Population was examined because it is growing and quality of life is increasing, both having carbon fueled economic growth implications. Because combustion is still the primary energy source that drives global economies and the type of CO_2 that has been increasing from the beginning of the industrial age is from combustion CO_2, CO_2 continues to increase (Jacobsen, 2018). Two future articles will examine 1) consequences of climate change at varying global temperature changes and 2) deploying solution technologies, practices and policy to avoid climate catastrophe.

I argue that accounting for abrupt change explains global warming more comprehensively. Specifically, the nonlinear model tested in this work focuses on three variable intersections across a 3D surface (CO2, population and temperature). In addition, the model takes an interdisciplinary approach by relying on thermodynamic law to predict change of state across the model surface. The model approximately characterizes a cusp catastrophe (Thom, 1975). In physics, biology, social science, including economics, linearity is more the exception, and nonlinearity is more the rule as expected by ubiquitous sudden change in virtually every scientific discipline. The use of thermodynamic law to explain a sudden change resulting from a social system may be somewhat rare. Nonetheless, the attempt is scientifically sound.

The theoretical study of a phenomenon most often centers on the use of a model. The model is supposed to closely mimic the features of the situation under investigation. For example, when we use Google maps to find a location on our phone, we use a digitally scaled down replication of some topological vicinity. We expect to find a very precise location from this display of quantitative information presented to us on something as small as a handheld device. The image we see is not a series of lines nor are the directions straight lines and if we select the satellite view, we see a nonlinear 3D version of our vicinity of interest that is closer to the reality of what we are investigating. We want to represent the real world as exactly as possible so it is more useful to us.

Linearity

The linear model is a nice way to acquaint ourselves with modeling however, nature doesn't align itself just for our convenience. The current study tests a linear and a nonlinear model. The linear "one-size-fits-all" approach to understanding the real world has obvious misgivings. The notion of $y = ax + b$ has dominated science for the past 150 years. Based on this premise, the central limit theorem took hold where it is believed by many that the Law of Large Numbers is the backbone of the theory of probability. Sufficiently large collections of similar quantities of variables are assembled into an ensemble and that every such ensemble takes on this universal property. Irrational researchers are quick to claim they have the tools to make estimates of probability and that most physical and social properties take on this simple characteristic and therefore they hold the wondrous world of prediction at their fingertips, so they assume (Jacobsen, 2010).

Nonlinear Dynamics

The notion of linearity and randomness arises in various guises in a number of distinct contexts and these forms are somewhat equivalent at a certain level of abstraction. Nonetheless, nonlinear dynamics is known to model most phenomena better and explain some systems more closely than others by virtue of letting the data speak louder than a standard prescriptive model where one size fits all. It should be evident that from rates of growth to stock markets to criminal behavior to change of state there is a vast body of nonlinear dynamics at work. Nonlinear dynamics is the study of phenomenon that are described by nonlinear equations in motion. Nonlinearity is the equation portion of this definition while dynamics implies motion and of course time is relative to motion. In this article, we are examining a well-known nonlinear equation that is only one of many popular scientific models. Self-organized criticality, fixed point attractors, catastrophe, chaos, fractals and limit cycles are but a few examples from the nonlinear songbook, so to speak. Indeed, there are an endless number of nonlinear models and each has distinct characteristics. Let us briefly look at a familiar example of an ubiquitous yet simple nonlinear model before moving on to the our nonlinear model for global warming.

Zipf's Law (1932), made inroads to explaining many extreme phenomenon with an equation so simple that it could have easily gone unnoticed. Zipf said that where

F is the frequency of something occurring and r is its rank, i.e. its relative order of occurrence (first = most often occurring, second = second most often occurring etc.…):

$$F \approx 1/r \tag{1}$$

Simply put, the frequency of an event F, is inversely proportional to its rank. Thus, the third most common word in the English language ranks third in usage, i.e. is found a third as often as the most common word. The fourth most common word in the English language is used one fourth as much as the most often used word and so on.

A number of authors have taken Zipf's concept and changed the equation with newer nonlinear models such as power laws, scale free networks and even fractals. The idea is that when an underlying structure exists where magnitude is a function of frequency on a double log plot a negative sloped line emerges. However, the Zipf equation only works where the behavior has these specific characteristics. Let's move on to another nonlinear model that likely mimics global warming.

Cusp Catastrophe Model

Catastrophe theory is a relatively new mathematical method used to describe an evolution of forms in the natural and social world. While investigating catastrophe theory one quickly learns that Rene Thom is endorsed as the originator and his book Structural Stability and Morphogenesis (1972) expands the philosophy. There are tell-tale signs of a catastrophic condition that lets the investigator know when to test for a cusp catastrophe. One such characteristic and perhaps the hallmark of catastrophe theory is when gradually changing forces result in sudden effects. Thom (1975) developed seven catastrophe models and each has different characteristics. These seven catastrophes have algebraic representatives given by polynomials in Table 1.

While we have examined a standard normal linear system, we have not accounted for the physical characteristics of global warming relative to the ice-water change of state. The most commonly found states are solid, liquid and gas. There are six changes of the three states: 1) freezing, 2) melting, 3) condensation, 4) vaporization 5) sublimation and 6) deposition. Our study is concerned with solid to liquid at 0 degrees C. Melting absorbs latent heat and latent heat absorption cools the surrounding (ambient) air while latent heat release warms the surrounding air. Presently, 10 percent of land area on Earth is covered with glacial ice, including glaciers, ice caps, and the ice sheets of Greenland and Antarctica. Glacierized areas cover over 15 million square kilometers (5.8 million square miles). The amount of water locked up in ice and snow is only about 1.7 percent of all water on Earth, but the majority of total freshwater on Earth, about 68.7 percent, is held in ice caps and glaciers.

	the *simple minimum*	x^2
1.	the *fold*	$F(u,x)= x^3 + ux$
2.	the *cusp*	$F(u,x)= x^4 + ux^2 + ux$ ($y = x^{2/3}$ *is a cusp too)*
3.	the *swallowtail*	$F(x,u,u,w)= x^5 + ux^3 + vx^2 + wx$
4.	the *butterfly*	$F(x,u,v,w,t)= x^6 + ux^4 + vx^3 + wx^2 + tx$
5.	the *elliptic umbilic*	$F(x,y,u,v,w)= x^3/3 – xy^2 + w(x^2 + y^2)- ux – vy$
6.	the *parabolic umbilic*	$F(x,y,w,t,u,v)= y^4 + x^2y + ux^2 + vy^2) + wx – ty$
7.	the *hyperbolic umbilic*	$F(x,x)= x^3 + y^3$

Tab. 1. *Seven catastrophe models as developed by Thom (1972)*

The cusp catastrophe model (see figure 1) has a set of characteristics such as bimodality, divergence, catastrophe, hysteresis and inaccessibility. A short description of each of these characteristics follows before moving on to an application.

Bimodality

For values of the predictor variables within this area of the cusp, the dependent variable can take on two values. This is different from a standard normal response surface that will have a single value as are more typically found in economics, for example. However, the system will not present or exhibit both forms of behavior at the same instant. The current state of the system is determined by its previous location on the surface and is a state-determined system. If the predisposition toward the next point results in markedly different behavior, then this aspect of the model is extremely useful. The model portrays this phenomenon by bifurcating or splitting (Oliva, 1991).

Divergence

As the magnitude of the bimodal or splitting factor (*u*) is increased, small initial differences in *v* may result in completely different trajectories, opposite at times, along the *x*-axis (equation 2). Therefore, a slight difference in the initial starting point may result in a very different behavior of the system other than what may be expected.

If the value of the splitting factor is increased, the points move forward on the surface, following roughly parallel paths. For most locations of these two points along the normal factor dimension, the points will continue to parallel each other by both moving onto the upper or lower surfaces of the fold. If they do, the difference in value of the predictor variable represented by the two points will continue to be slight. However, if the forward movement of the two points leads one to the upper and the other to the lower, then corresponding values of the predictor variable will diverge. The slight initial difference in the normal variable will lead to an increasing difference in the dependent variable as the splitting factor increases (Biglow, 1982).

Incidences of sudden, discontinuous changes along a dependent variable dimension are probable in many different branches science. They occur when the values of the independent variables are within the area of the cusp, and the normal factor is increased in the direction of the opposite surface such that it crosses out of the cusp. Travel on the folded part of the surface can result in a sudden shift from the bottom sheet and vice versa. The surface can model situations that may exhibit evolutionary and revolutionary forms of behavior, as in the change of state, splitting an atom, adoption of disruptive technical advance or a dog going into rage. Evolutionary and revolutionary behaviors occur along the same surface, but at opposite sides.

Evolutionary behavior is where low values of the splitting factor and changes in the normal factor lead to proportional corresponding continuations that are considered non-catastrophic with respect to the dependent variable. There are predictable changes that are gradual, expected and non-disruptive along the surface until the critical value intersection.

Revolutionary change is where the normal and splitting factor lead to a path that crosses the bifurcation set. This is an abrupt behavior where catastrophic change in the value of some stage continuum (dependent variable) occurs. This path follows the lower surface, enters the bifurcation set, and runs off the edge of the fold at the other side. This is the point where an abrupt transition is made from the lower to the upper surface. This situation may reverse, but not at the point in the value continuum where the first change occurred; rather and most often, it will take considerably more to punch it back to drop down to the lower surface.

Hysteresis

Once the transition is made from one part of the surface to the other within the folded area, a return to the values of the independent variables at the point of catastrophe will not cause the system to return to its original dependent variable value. Normally, the system does not shift back as it would as if it were a simple step function. This property, whereby a second catastrophic change, once made, requires substantial reversal of the normal factor to undo, is referred to as hysteresis. Magnetism is characterized in this way.

Inaccessibility

In the region of bimodality, the surface folds under, forming a third layer between the two modal values. While this completes the deformed surface by connecting the upper and lower surfaces, the dependent variable does not take on values represented by the surface. For the purposes of this study, it is relatively inaccessible, and this region is at times excluded from catastrophic graphics entirely. However, the mathematical representation of the surface has a middle sheet that completes the pleat (Guastello, 1995 & 2002). In terms of the theory it represents, it is the area of the least probable behavior. Therefore, for the practical theoretical exposition of this study, the dependent variable cannot take on these values from changes in the independent variables. This is the change of state where temperature change can slow, stop or drop due to the extra energy needed to change state at 0 C at atmospheric pressure, for example.

Cusp Catastrophe Graphic Display

The cusp catastrophe is a universal unfolding of the function $F(x) = x^4$. Its equation is:

$$F(x, u, v) = x^4 + u * x^2 + v * x \tag{2}$$

We want to look at the catastrophe surface, that is, the set of singular points. Those are the points in (x, u, v)-space, where the partial derivative of F with respect to the system variable x is zero. As we can see in figure 1, there is a cusp surface in 3-space.

Figure 1 is similar to a cusp catastrophe model that was developed by Sanns (2000) while using Mathematica, Version 4.0. I used Mathematica version 5.0 to produce it. In appendix B, you will also find the syntax used to display the cusp catastrophe model and similar procedures are used for other three-dimensional plots in Mathematica today (version 11).

Thom (1972) was not the only one to examine the sudden change phenomenon. The A represents the asymmetric variable and the B represents the bifurcation variable while temperature is the dependent variable in Figure 1. Abrupt change has been characterized and documented by scientists throughout the history of physics, biology and social science. Earthquakes, asteroid impacts, avalanches and buckling beams are a few in the physical sciences. In biology a some are strokes, heart attacks, as well as the spreading and cures for disease. In social science there are many such as riots, recessions, consumer behavior and addiction. Models and concepts used that have this tipping point characteristic as a central theme are self-organized criticality (Bak, 1996), punctuated equilibrium (Gould, 2007) and fractals (Liebovitch, 1998). Here I examine the physical change of state, all occurring at zero degrees C or 32 degrees F. Let us list the physical thermodynamic mediation.

Three Mediators at Zero C

1) Change of state and heat absorption change
2) Frozen and thawed rotunda containing methane
3) Change of color (near white to near black)

All three of the items listed above operate simultaneously. Yet, they do not start nor complete their cycle of state change at the same instant. For the purposes of this study, when temperature reaches zero C over time, we begin the new state condition. Let us move on to our methods.

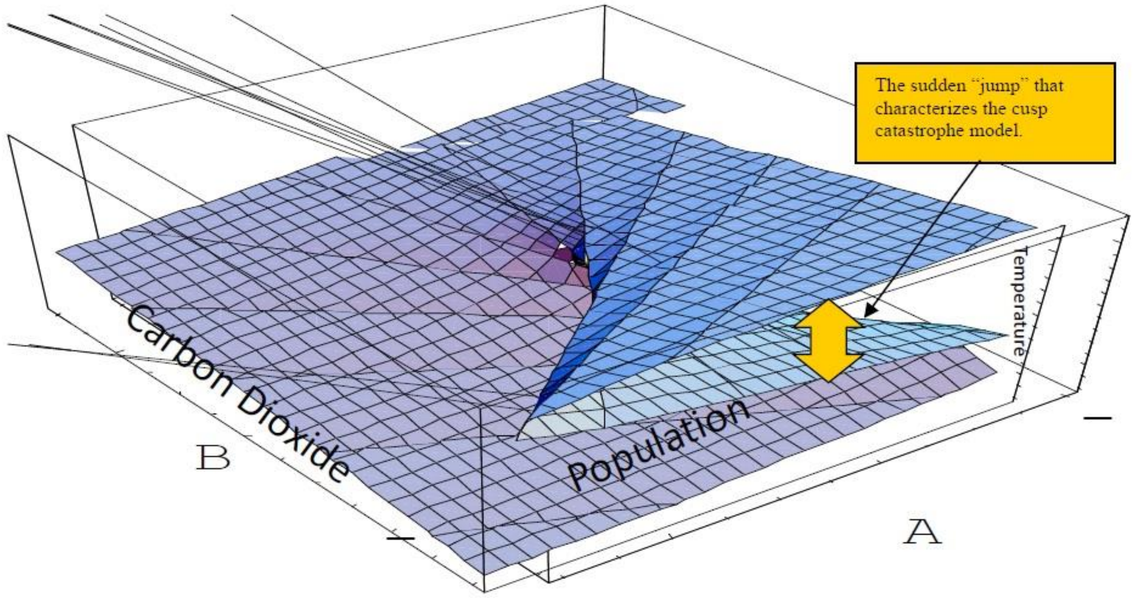

Fig. 1. *Cusp Catastrophe Graphic Model - Sudden Jump*

METHODS

Three variables (two independent and one dependent) are used to test a linear model. The linear model is set as earth temperature as a function of CO_2 and population. A second is a nonlinear model referenced as a cusp catastrophe and proposed with the same variables. However, the nonlinear cusp catastrophe model considers the physical changes and conditions of the environment. The interdisciplinary (demography, economics and physics) approach to modeling is not new but somewhat rare. The advancement of predictive analytics helps to push boundaries and this kind of departure from the standard normal model may be refreshing. Three primary data sources were used for the variables: 1) population, 2) CO_2 and 3) earth temperature and all aligned for annual observations on a spreadsheet. Population data is from the United Nations Department of Economic and Social Affairs (2017) and is measured as per/person alive allowances for births and deaths as a differential in the given year. The United Nations department releasing the reports and spreadsheets is part of the World Population Prospects: The 2017 Revision of the Department of Economic and Social Affairs.

Carbon dioxide data was collected by the National Carbon Dioxide Information Analysis Center (2017), Oak Ridge National Laboratory, US Department of Energy (DOE). I also confirmed with data from NASA, Goddard Institute, (2017), Earth Policy Institute, WorldWatch Institute, Signposts; from 1960 to 2017 and lastly NOAA/ESRL, Atmospheric Carbon Dioxide - Mauna Loa. Units of measure are annual in parts per million (PPM) and smart sensors are used that assure sensor calibration feedback loops and data point transmission to collection servers for direct reports.

Data used for earth temperature was NASA, Goddard Institute, (2017), Earth Policy Institute, Worldwatch Institute, Signposts; from 1960 to 2017 and lastly NOAA/ESRL, Atmospheric Carbon Dioxide - Mauna Loa all warehouse temperature. The units of measure are measured in annual observations and the actual units are of temperature are in Centigrade not Fahrenheit. Smart sensors are used that assure sensor calibration feedback loops and data point transmission to collection servers for direct reports.

Interdisciplinary Approach to Modeling – three tipping point forcing

I'll briefly review three physical forcing variables where each presents a potential impact on global warming. The interactions of the three forcing factors or mediators described below compound the effects because they are happening concurrent relative to zero C and all are a consequence of the change of state phenomenon. The change of state process is not a candidate for dispute.

Fundamentals of Heat: change of state

To more fully explain the physical properties of global warming we must consider a tipping point where thermal relative discontinuations occur (Lemoine, D., & Traeger, C., 2014; Lenton, T., 2012;

Wassmann, P., & Lenton, T., 2012). At zero C (32 F) we have an interruption to the steady proportional variation across the temperature distribution. At atmospheric pressure (14.7) and below zero C ice requires .47 Btu to increase the temperature of one pound one degree F, therefore the specific heat of ice is .47. The specific heat of water is one. It therefore takes 1 Btu to increase the temperature of one pound of water one degree F and not coincidentally this is the definition of a Btu (Solids, Liquids and Gasses, 2019).

Linearity is only relative to a single state system across the energy input continuum as related to the temperature change output continuum. Once the energy input and respective response temperature continuum output reaches the linear physical state limit (see figure 2 below) energy input begins the change of state at zero C, pressure variations notwithstanding. Solid water (ice) given the appropriate amount of heat turns into liquid water. During the transition, it takes 306.38 (144 Btu per pound) times more heat energy (than .47 Btu) to change the state from ice to water (solid to liquid) at 0 C (Mochizuki & Koga, 2015).

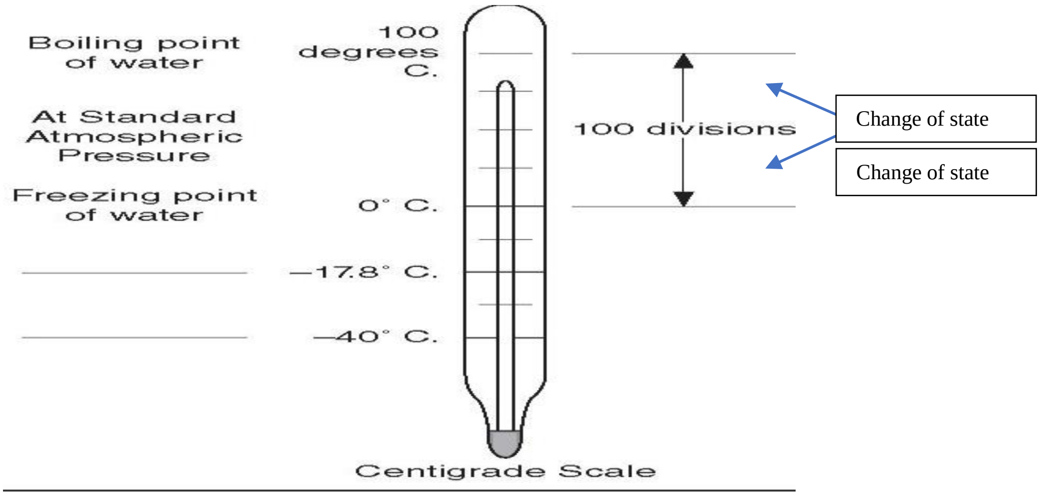

Fig. 2. *The temperature state continuum for water is defined*

Within the 100 divisions from 0 to 100, it takes 1 Btu to change one pound of water 1 degree F up or down creating a perfect linear symmetry and this is referred to as sensible heat. Moving outside this region departs from the symmetric proportion in variation across the water temperature/heat energy relationship. To cross the high side threshold at 100 degrees it takes 970 Btu of heat energy to change one pound of water to one pound of steam before a change in temperature. To reverse from one pound of steam to one pound of water 970 Btu is given up from the steam while the steam condenses to water (Jacobsen, 2011). On the low side it takes 144 Btu release to change one pound of water to one pound of ice. Conversely, it takes 144 Btu of heat absorption when ice changes into liquid water, an equivalent amount of heat but it is absorbed (144 Btu per pound). The phase change releases heat (exothermic) when converting gas to liquid or liquid to solid. The phase change absorbs energy/ heat (endothermic) when going from solid to liquid or liquid to gas.

We cannot ignore the physics of state change in global warming and therefore we must account for the increased heat energy required to change state (Cockerell, 1902). As the poles and ice caps melt, we find ourselves in the change of state region today. Once the change of state is complete, there is excess heat energy availability because not only is ice heat absorption over, the heat energy source is not decreasing but is increasing so ambient temperature will therefore increase.

Change of Color

Changing color from white snow to near black sea water is the second critical point along the temperature continuum. The albedo of a surface is measured on a scale from 0 to 1, where 0 is a idealised black surface with no reflection, and 1 represents a white surface that has perfect reflection. The roofing industry takes advantage of the principal by producing white roofing systems that assist cooling systems so they use less energy to maintain comfort, especially in hot climate zones. The most significant projected impact on albedo is through global warming. Nearly all the ice on the planet is melting. As the white surfaces decrease in area, less energy is reflected into space, and the Earth will warm up even more. The loss of Arctic ice is of particular concern. The ice is disappearing quite fast; not only is albedo decreasing, but the loss triggers a positive feedback. By exposing the ocean surface to sunlight, the water warms up. This melts the ice from underneath, while man-made CO_2 in the atmosphere warms the surface. The albedo change amounts to 25% of mediation from CO_2 over a 30 year period (Pistone, Eisenman & Ramanathan, 2014). Humidity also increases; water vapor is a greenhouse gas. More ice therefore melts, which exposes more water, which melts more ice from underneath…and the cycle continues. This is a good example of a positive feedback. Increased water vapor also has another effect, which is to increase the amount of cloud. Clouds can increase albedo (a negative feedback), but also warming (a positive feedback). However, if the receding white surfaces are replaced with vegetation, the CO_2 absorption may slow the increase in heat (Bright, Bogren, Bernier & Astrup, 2016)., not likely to be a very quick offset at or near frozen ice.

Thawing Permafrost

The third critical change is the thawing of the deep-sea soil and water permafrost.

Nobody knows exactly how much carbon is housed on the boreal and alpine permafrost. Estimates are in the vicinity of 350 to 400 gigatones. This could be as much as a quarter of all carbon found in soil on earth. Microbial decomposition of previously and currently frozen organic carbon is both slowly and abruptly mediating feedback loops from past ecosystems to further the greenhouse effect in global warming today. Other aspects of ecosystem dynamics can be altered by global warming along with thawing artic and boreal permafrost. Previous research considered cold soil as a carbon sink. However, these processes do not appear to be able to compensate for the extensive release from thawing so it certainly qualifies as a continual feedback loop increasing the release over time (Schuur et al., 2008). Methane is also a very serious threat and the other greenhouse gas released similarly from the sea floor at the change of state intersection. Let us move on to linearly modeling the three variables and nonlinear modeling the cusp catastrophe model with an attempt to capture the physical changes.

Analysis

Three variables are considered: earth surface temperature CO_2 in ppm and finally population of the earth. Two general models are built and explained. The first is a time series multivariate linear model that considers historical publicly available data. The second is a nonlinear multivariate model but considers a cusp catastrophe where the sudden change (region of least probable behavior) is temperature precipitated by the physical mechanics of state change.

For the linear equation, equation 3 formalizes temperature as a function of global population and atmospheric CO_2 in PPM.

$$\hat{y} = b_0 + b_1X_1 + b_2X_2 \quad (3)$$

where B_0 is the intercept, B_1 is the coefficient of X_1 (population) and B_2 is the coefficient of X_2 is CO_2 in PPM.

Cusp

There are critical points where rapid change takes place. For values of the predictor variables within the area of the cusp, the dependent variable can take on two model values according to the function:

$$\frac{df(y)}{dy} = Y^3 - by - a, \quad (4)$$

which describes a continuity that underlies the changes between the two stable states. The cusp catastrophe model appears in Figure 2 with respective labels for the application to climate change.

The second model proposes the cusp catastrophe as developed by Thom (1972). This model is formalized as equation 5 below. The dependent variable is temperature as a function of global population and atmospheric CO_2 in PPM. The cusp interruption is proposed as the change of state at or nearing completion.

$$F(x, u, v) = x^4 + u * x^2 + v * x \quad (5)$$

Input syntax for the cusp catastrophe 3D-model in Mathematica, as developed by Sanns (2000) is found below.

$F(x, u, v) = x^4 + u * x^2 + v * x.$

```
Remove["Global`*"]
F[x_,u_,v_]:=x^4+u*x^2+v*x
D1F[x_,u_,v_]:= Evaluate[D[F[x,u,v],x]]
<<"Graphics`Master`"
CriticalPoints=ContourPlot3D[D1F[x,u,v],{u,2.5,2},{v,2,2},{x,1,1},PlotPoints→,
ViewPoint→{2,1.5,2},Axes→True,AxesLabel→{"u","v","x"},DefaultFont→14]
```

Findings

Results from analyzing our three variables from a linear perspective yields some interesting results. If the distributions of the three variables continued on their respective linear path without disruption, we may be able to rely on these results. These results are possibly useful to predict future conditions within limits. Below in Table 2 we have a standard normal linear multiple regression model where the response variable is temperature and the predictor variables are world population and earth CO_2 in ppm. There are 68 annual observations starting with 1950. The time period spans from post economic adjustment years just after WW2 when the US economy had global dominance.

Regression Statistics				
Multiple R	0.936573106			
R Square	0.877169183			
Adjusted R Square	0.873389773			
Standard Error	0.093185923			
Observations	68			
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	8.760413722	0.945642	9.263983	1.70257E-13
World Population	-1.81174E-10	6.87E-11	-2.63753	0.010437309
CO_2 ppm	0.018195783	0.003663	4.968092	5.18871E-06

Tab. 2. *Linear regression output for temp as a function of pop and CO_2*

For every one unit change in ppm (one ppm) we see a .018 change in earth temperature. These results are significant to the .0000052 level. When simultaneously considering population as another independent predictor, for very 55.55 ppm we have significant support for the increase of 1 degree C. For every one unit change in population (one person) we see a -.00000000018 change on temperature. As population increases, we are seeing a very slight reduction in CO_2. This may seem counterintuitive yet considering the interactions of the two predictors, many explanations emerge. Interactions of independent variables reflect the variations among independents and mediation of the dependent. If one independent is more tightly bound to the fluctuation in the dependent, other independent variables may be overshadowed. Nonetheless, for every additional unit of CO_2 the model suggests a highly significant change in temperature. Approximately 87% of the linear model fits the data. According to our linear model, if there were no population and no CO_2, the temperature would be 8.76 degrees C. Lastly, the model suggests that with a condition of zero population and zero ppm of CO_2, the temperature would be 8.76 C.

Cusp Catastrophe Model of Population, CO2 and Temperature

Let us now use the cusp catastrophe to model our problem with a little help from thermodynamics. It takes an additional 970 Btu to turn one pound of water into a 1 pound of steam at 100 C (212 F) with no change in temperature. It requires an additional 144 Btu to melt one pound of ice to one pound of water at 0 degrees C (32 F) with no change in temperature. Therefore, depending on how many pounds of ice exist, temperature change will actually slow at 0 C while change of state takes place. Let us return to the cusp figure to graphically display the time relative sudden jump:

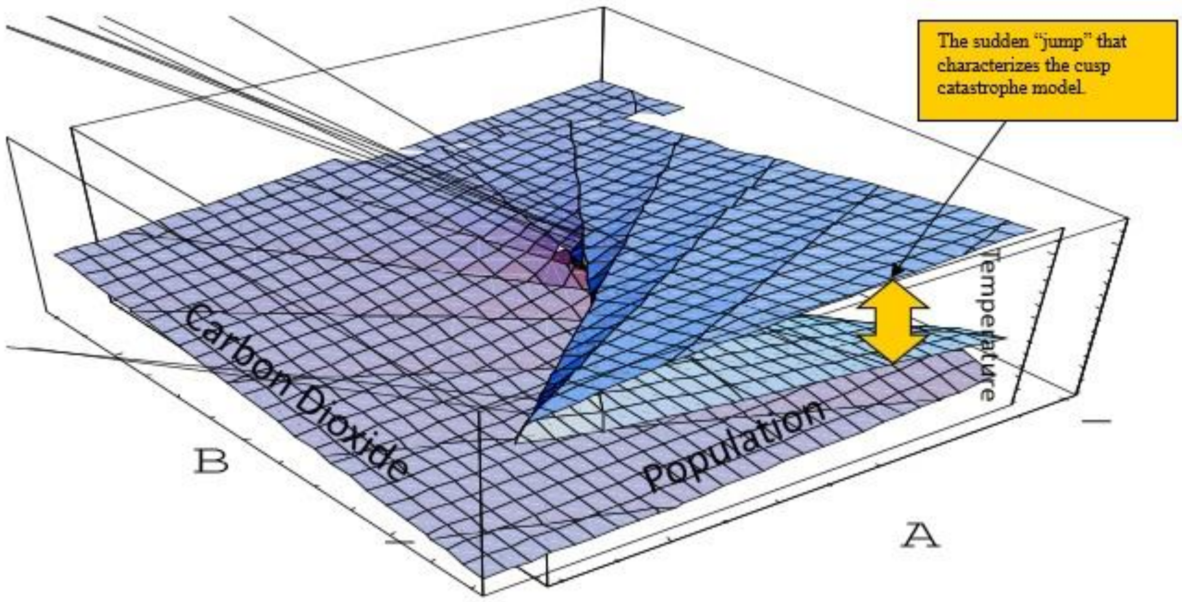

Fig. 3. *The cusp catastrophe model demonstrates a steady state behavior for a determined interval*

Population is relatively predictable. As the two predictor distributions approach the lip of the cusp from the lower surface, no change in the rate of change is relatively meaningful. In fact, at the lip of the cusp, temperature change stagnates, slows or cools (depending on proximately factors) due to the increased energy absorption required to change state (solid to liquid) at zero C at different times for different geographical locations, resulting in less temperature change, variance and speed of change of state, i.e. increased state change - decreased temperature change, given fixed income energy. Not all ice will melt at the same moment but, if it did, the thermal absorption would send the nearby system into a sudden drop in temperature. Lucky for us there is a gradual reduction in the rate of change in temperature increase because of the steady solar energy needed to do the hard work required to change state at different locations at differing speeds over time. Unlucky for us is once the state change is sufficiently complete, ambient temperature change will rapidly accelerate because of the 1) increased available energy, 2) increased CO_2 and methane release from the permafrost and change in albedo i.e. cusp entry.

Cusp

The cusp model for temperature change:

$$pdf(z) = \xi \exp[\theta_1 z^4 + \theta_2 z^3 + \theta_3 z^2 b + \theta_4 az]; \quad (6)$$

Where *pdf(z)* is the cumulative probability of the temperature, ξ and θi are nonlinear regression weights, *b* is the standardized CO_2 ppm, and *a* is standardized population. Nonlinear regression weights should all initialized with a value of 0.5, *z* is standardized temperature (equation 6).

In Equation. 7, values are standardized as:

$$z = (y - \lambda)/s \tag{7}$$

Asymmetry (*a*) and bifurcation (*b*) were also transformed according to Equation. 7, such that the lowest value of each variable (location, λ) was added to each value of the original variable resulting in all positive numbers. Each of the new *z* values was further transformed to the fourth, third, and second power, as shown in Equation. 3. The principle objective of greenhouse gas emission reduction is climate stability. Therefore, the single most important outcome we can hope to achieve in our emission reduction efforts is to avoid the sudden jump behavior (accelerated increase in temperature) that is going to happen if we fail to meaningfully reduce fossil fueled combustion.

The current state of the system is determined by its previous location on the surface and is a state-determined system. If the pre-position points result in markedly different non-proportionate outcome temperatures, then this aspect of the model is extremely useful. The model portrays this phenomenon by bifurcating (Oliva, 1991). A qualitative change in a dynamical behavior of a system or a topological structure of its phase portrait as parameters move across a critical point is a bifurcation, which is unstable and in this case harmful. The assembly of all such points is the bifurcation set (Sprott, 2003) and exceedingly crucial to identify. A linear model would not detect this behavior. These important determined subsystems could easily be dismissed as random systems unrecognizable to the investigator. It is important to know where bifurcations occur in order to produce desirable outcomes or avoid undesirable outcomes, i.e., they are scientifically useful (Jacobsen & Guastello, 2007).

DISCUSSION

The implications of global warming could not be more important. Even though the relationships among combustion CO_2, population and Earth temperature are meaningful and significant (Jacobsen 2011), the entire picture is not revealed without an account for the physical thermal changes that take place at 0 C. In this study, I attempt to simplify a topic that might be viewed as overly difficult. It is suggested that people may dismiss or even avoid what they perceive as too complex. People may also ignore things because they feel they can't make a difference anway. However, I believe the subject of global warming is too important to ignore. I may assume that everyone understands something about it and everyone has discussed it but maybe not some specific more scientific details. It is in this sentiment that prompted me to run a few models that are part of the global warming picture. Let us look at a few implications. Likely to be the most salient and impactful outcome from global warming is sea level rise. Calculating glacier ice volumes and sea level equivalents is a straight forward set of multipliers. Sea level equivalent for a given volume of ice is convertible. A first-year engineering student could easily calculate the conversion. Density equals mass divided by volume, mass equals volume times density and volume equals mass divided by density. The volume of ice is given in km^3 while the mass of ice (at this scale) is given in gigatonnes. The conversion is a simple equation:

$$\text{Mass of ice (Gt)} = \text{Volume of ice (km}^3\text{) x density of ice (Gt/km}^3\text{)} \tag{8}$$

The mass of ice is 458 Gt and if all melted it would render 458 Gt of water. Just as an example, let us briefly look at the Antarctic. Bedmap1 and the improved Bedmap2 (Fretwell et al. 2013) thickness grid (about 25 million measurements) at over two orders of magnitude more than Bedmap1, revealed mountains, valleys, basins and troughs where only fragments were previously revealed. Bedmap2 shows the volume of the Antarctic ice sheet to be a mere 27 million km^3 with a sea water conversion of 58 m or 190 feet sea-level rise. Interesting enough, this is the same result Bedmap1 (2001) found. We are set to reach a global population of 11.2 billion by 2100 and most of that growth is in developing nations where most are moving to new and existing mega cities in costal zones. Table 3 below specifies growth for every city where Tokyo is the only metropolitan area to decrease in population.

In terms of the economic implications, considering climate driven feedback and the increased speed of warming, it should be no surprise that it is and will continue to be costly. Rising seas and stronger storms will cost plenty of money and this is not the kind of GDP we want. The accelerated global warming and associated feedback loops may add $70 trillion to the overall cost (Leahy, 2019). After meeting the Paris targets the $70 trillion is the estimate. If the agreement is not met, it will certainly increase the estimate. The good news is that if we can limit warming to 1.5 C (2.7 F) the cost of Arctic warming falls to about $25 trillion. Leahy points out that global GDP for 2016 was $76 Trillion just to get some perspective on these rather large numbers. With so many countries shifting to cleaner technologies and practices, a low carbon economy is the next big economic powerhouse. Energy is the most important driver of our economic systems (Jacobsen, 2011). I could easily argue that energy is becoming more important as we become engulfed in more people, more global markets, automation, AI, big data and machine learning because they are all a function of energy. Virtually every aspect of our survival is brought to us via energy. Yet, we risk national security and well-being by relying on polluting, politically charged nonrenewable energy with limited availability and eventual depletion. The shift to more reliable, stable and renewable energy powered systems is necessary because we have created a world where humans cannot survive without abundant energy.

Urban Agglomeration	2015	2020	2025	2030
Tokyo	38 001000	38 323000	37 876000	37 190000
Delhi	25 703000	29 348000	32 727000	36 060000
Shanghai	23 741000	27 137000	29 442000	30 751000
Beijing	20 384000	24 201000	26 494000	27 706000
Mumbai (Bombay)	21 043000	22 838000	25 207000	27 797000
Dhaka	17 598000	20 989000	24 331000	27 374000
Mexico City	20 999000	21 868000	22 916000	23 865000
São Paulo	21 066000	22 119000	22 899000	23 444000
Al-Qahirah (Cairo)	18 772000	20 568000	22 432000	24 502000
Karachi	16 618000	19 230000	22 009000	24 838000
Osaka	20 238000	20 523000	20 348000	19 976000
Lagos	13 123000	16 168000	20 030000	24 239000
New York-Newark	18 593000	18 793000	19 314000	19 885000

Tab. 3. *UN projections for major urban centers*

Future Research

It may be possible to circumvent this apparent catastrophe. However, it involves three major changes in how we 1) design and operate our logistical, residential, commercial and industrial economies and 2) accelerate the diffusion of clean energy systems and other clean technological developments. Unlike a standard linear model or the normal S-curve of the diffusion (cumulative adoptions) of innovations, the sudden shift in Earth's surface temperature from CO_2 increases over a short period of time suggest that the distribution of clean technology adoptions also requires a sudden shift in the adoption of clean technology. It is suggested that within the energy intensive industries that more research is needed involving the accelerated adoption of clean energy technological advances. In addition, sudden changes in design and operation of our logistical, residential, commercial and industrial economies should also conduct more research. Both of these suggestions are made in order to offset the sudden escalation in temperature according to the United Nations (2018) latest report. It is suggested that economic scholars conduct more research on the financial implications of global warming. For business and sometimes government, the economic costs of global warming are of more concern than other negative outcomes. There is a need for continuous updating the research on very specific economic details concerning health, migration patterns, and water and food availability.

Lastly, research is needed in the history, current state and the future of transitioning to new energy systems and especially a clean electrical grid. Given the current state of technology (without efficient DG/CHP and renewable dominance), flexibility (plug and play) and pricing structures (net billing and buy back rates) that create incentives for transition to clean energy systems. This is perhaps the most important because when the incentive for distributed generation is appropriate, distributed systems are installed such as efficient DG/CHP and renewables.

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Input syntax for cusp catastrophe 3D-model (Mathematica) developed by Sanns (2000)

Equation:

$F(x, u, v) = x^4 + u * x^2 + v* x.$

```
Remove["Global`*"]
F[x_,u_,v_]:=x^4+u*x^2+v*x
D1F[x_,u_,v_]:= Evaluate[D[F[x,u,v],x]]
<<"Graphics`Master`"
CriticalPoints=ContourPlot3D[D1F[x,u,v],{u,2.5,2},{v,2,2},{x,1,1},PlotPoints□,
ViewPoint□{2,1.5,2},Axes□True,AxesLabel□{"u","v","x"},DefaultFont□14]
```

Якобсен Дж. Дж.

Модель катастрофи зміни клімату з точкою повернення – супутник глобального потепління

У статті досліджується зміна клімату як міждисциплінарна експозиція динаміки економіки, динаміки народонаселення, термодинаміки й нелінійної динаміки. Застосовано модель катастрофи з точкою повернення, запропонованої Рене Ф. Томом (Thom, 1975), до опису зміни клімату. Представлена модель з точкою повернення має три глобальних змінних: 1) народонаселення, 2) еміссія CO2 і 3) температура. Вивчаються взаємовідношення на різних точках перетину моделі для зображення вірогідної інтерпретації кліматичних змін. Хоча стаття є теоретичною за своєю суттю, автор запозичує методи і підходи від природничих наук для пояснення раптової зміни.

Ключові слова: *зміна клімату, CO2, населення, катастрофа з точкою повернення, енергетичні системи.*

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СИСТЕМНІ ОЗНАКИ МЕХАНІЗМУ ДЕЦЕНТРАЛІЗОВАНОГО УПРАВЛІННЯ ПРИРОДНИМИ РЕСУРСАМИ

Метою статті є визначення системних ознак формування механізму децентралізованого управління природними ресурсами, адаптованого до європейських стандартів господарювання. Розроблено методологічні підходи до створення системи механізмів управління природними ресурсами в умовах децентралізації влади в Україні на основі принципів структурного та проектного управління, субсидіарності, належного врядування, публічно-приватного партнерства та управління природними ресурсами за участю місцевих громад, структурно-